

---

## Computer Controlled Systems

### Homework 3

Submission deadline: November 16. 2017. 10:00/12:00 (end of the seminar)

All solutions are expected to be calculated by hand, also all figures have to be drawn by hand. Computer programs can be used for self-verification, but all problems have to contain the detailed steps of solutions!

### Frequency response analysis and controller design for a HDD head<sup>1</sup>

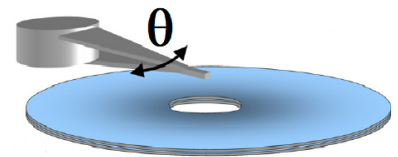
Consider a HDD read/write head with dynamics described by the following differential equation:

$$J\ddot{\theta} + b\dot{\theta} + k_1\theta = k_2u$$

where  $\theta$  denotes the angular position of the head, while  $u$  is the input.

The names and values of the system constants are:

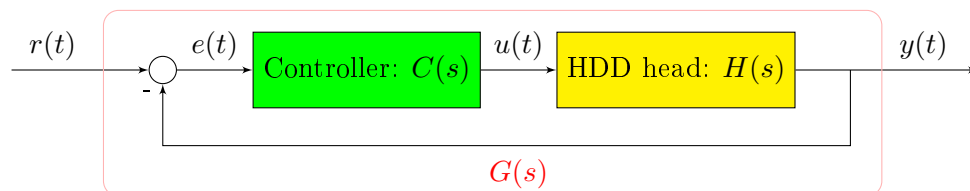
$J = 0.01 \text{ kgm}^2$	inertia of the head assembly
$b = 0.004 \text{ Nm/(rad/sec)}$	viscous damping coefficient of the bearings
$k_1 = 10 \text{ Nm/rad}$	return spring constant
$k_2 = 0.05 \text{ Nm/rad}$	motor torque constant



By choosing the angular position as output variable ( $y = \theta$ ), the input-output model of the HDD head is:

$$\ddot{y}(t) + 0.4\dot{y}(t) + 1000y(t) = 5u(t)$$

The head has a (damped) oscillatory behaviour, which is unacceptable. Moreover, it has to be positioned to prescribed values in a short time. To fulfill these needs, a PID controller is applied to the system as depicted in the figure below.



1. Determine the transfer function  $H(s)$  of the input-output model by Laplace-transform!
2. Check the poles of  $H(s)$ ! Is the system stable? What refers to the oscillatory behaviour?
3. Give the frequency response function  $H(j\omega)$  of the system! Give its gain  $k(\omega)$  and phase  $\phi(\omega)$  at  $\omega = 0 \text{ rad/s}$  and  $\omega = 32 \text{ rad/s}$ ! What is the response  $y(t)$  to the periodic input  $u(t) = 10 \sin(32t)$ ?
4. Give the transfer function  $G(s)$  of the closed loop system!
5. Determine the tuning parameters  $K_P, K_D$  and  $K_I$  in such a way that all the poles of the closed loop system are equal to  $-20$ !
6. Determine the DC gain of the closed loop system! Does this controlled system follow a constant reference (head positioning) signal?

---

<sup>1</sup>Model is borrowed from [MathWorks](#)

**Extra homework.** Implement and simulate the behaviour of HDD read/write head dynamics.

1. Implement both the controlled system ( $H(s)$ ) and the PID controller ( $C(s)$ ) as a “Transfer Fcn” (Figure 7, or as you want).
2. Be aware that, in case of a “Transfer Fcn”, the degree of the denominator must be at least the same as the degree of the numerator. E.g. instead of the improper transfer function  $\frac{s^2 K_D + s K_P + K_I}{s}$  use its semi-proper approximate  $\frac{s^2 K_D + s K_P + K_I}{\varepsilon s^2 + s}$ , where  $\varepsilon$  is small enough (e.g.  $\varepsilon = 0.001$ ).
3. Define a reference signal. You may use the “Signal Builder” block (Figure 5, Figure 8).
4. In the real life, an actuator cannot execute arbitrarily large input signal, e.g. a voltage source cannot generate arbitrarily high voltage. Therefore, before we connect the design control input computed by  $C(s)$  to the plant  $H(s)$ , we should limit the range of the input signal to the interval  $u(t) \in [-1000, 1000]$ . In order to implement this you can use the “Saturation” block (Figure 3).
5. Plot the response of the control loop alongside the reference signal (Figure 4, Figure 9).

You are allowed to solve the problem as you prefer. However, to compute a signal’s derivative, let me advise you against using the “Derivative” block (it is quite erroneous). Instead, use a semi proper transfer function  $D(s) = \frac{s}{\varepsilon s + 1}$ , where  $\varepsilon$  is small enough (e.g.  $\varepsilon = 0.001$ ) to give a proper approximate of the signal’s derivative.

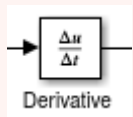


Figure 1.

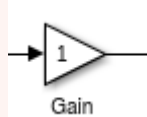


Figure 2.

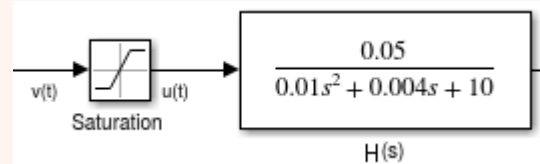


Figure 3.

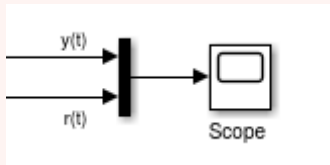


Figure 4.

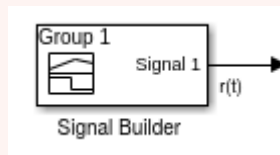


Figure 5.

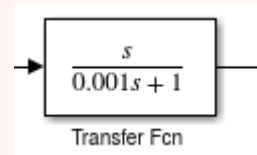


Figure 6.

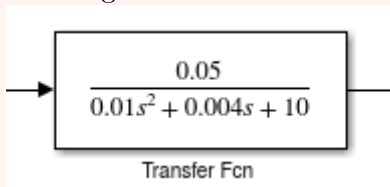


Figure 7.

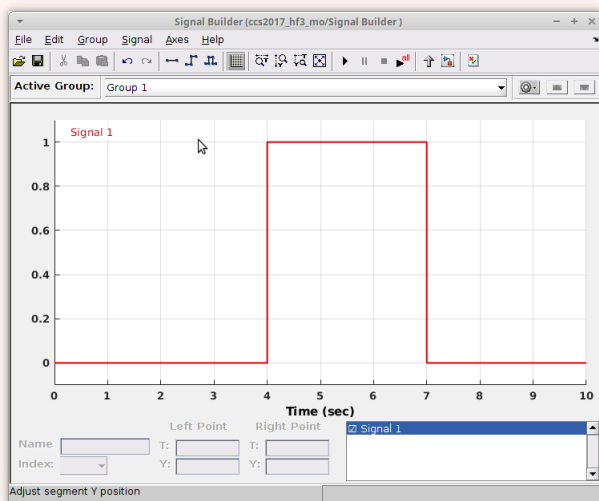


Figure 8.

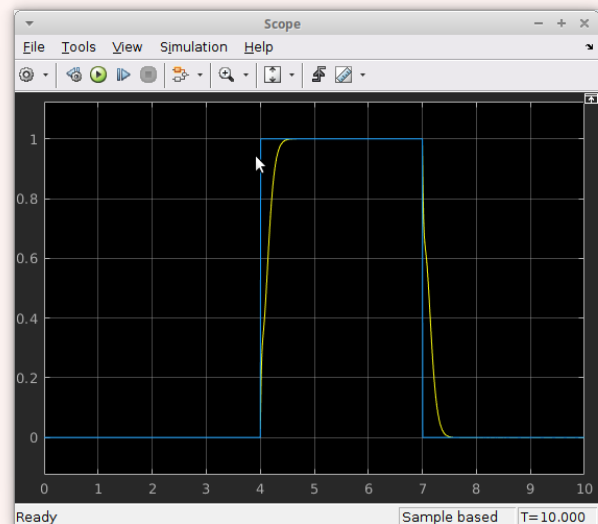


Figure 9.